2020

MATHEMATICS — GENERAL

Paper : DSE-A-2

(Graph Theory)

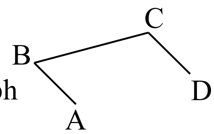
Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Day 1

1. Choose the correct alternative : 1×10

(a) The adjacency matrix of the graph 

is

(i) $\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 1 & 0 & 1 & 0 \\ C & 0 & 0 & 1 & 0 \\ D & 0 & 1 & 0 & 1 \end{array}$

(ii) $\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 0 & 1 & 0 \\ B & 0 & 1 & 0 & 0 \\ C & 0 & 1 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array}$

(iii) $\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 1 & 0 & 1 & 0 \\ C & 0 & 1 & 0 & 1 \\ D & 0 & 0 & 1 & 0 \end{array}$

(iv) $\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 0 & 1 & 0 \\ B & 0 & 1 & 0 & 0 \\ C & 1 & 0 & 1 & 0 \\ D & 0 & 1 & 0 & 1 \end{array}$

(b) A path has

(i) no repeated edges but repeated vertices

(ii) repeated edges but no repeated vertices

(iii) no repeated edges and no repeated vertices

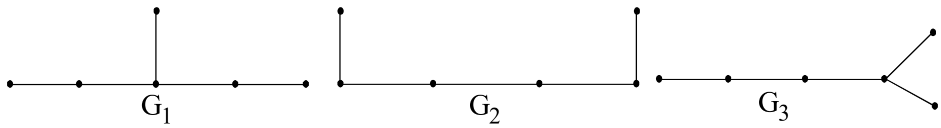
(iv) any of the above.

(c) The number of vertices of a regular graph of degree 4 with 10 edges is

(i) 3 (ii) 4 (iii) 5 (iv) 6.

Please Turn Over

(d) For the given graphs G_1, G_2 and G_3



(i) G_1 is isomorphic to G_2 and G_2 is isomorphic to G_3

(ii) G_1 is not isomorphic to G_2 and G_2 is not isomorphic to G_3

(iii) G_1 is isomorphic to G_2 but G_2 is not isomorphic to G_3

(iv) G_1 is not isomorphic to G_2 but G_2 is isomorphic to G_3.

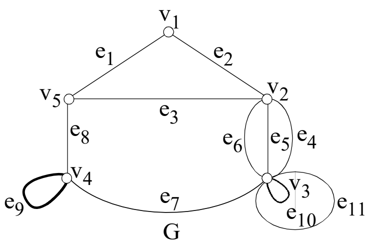
(e) Maximum number of edges in a simple graph with 11 vertices is

(i) 10 (ii) 22 (iii) 55 (iv) 110.

(f) A complete bi-partite graph $K_{m,n}$ has a Hamiltonian circuit if and only if

(i) $m = n$ (ii) $m = n + 1$ (iii) $m = 2n$ (iv) $m \neq n$.

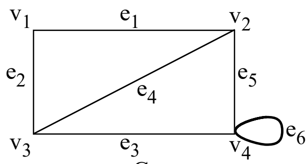
(g)



Adjacency matrix of G has how many 1's?

(i) 0 (ii) 5 (iii) 10 (iv) 12.

(h)



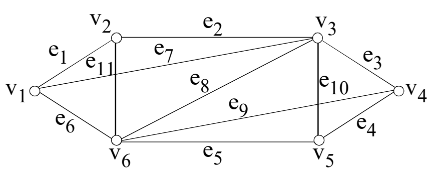
Number of walks of length 2 from v_2 to v_3 in G is

(i) 0 (ii) 1 (iii) 2 (iv) 3.

(i) Number of vertices in a tree with degree sequence {5, 4, 3, 1, 1,..., 1} is

(i) 8 (ii) 9 (iii) 10 (iv) 11.

(j)



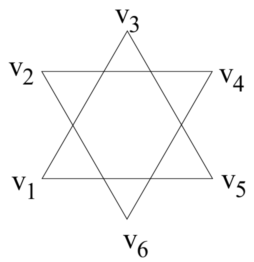
G is G

(i) both Eulerian and Hamiltonian (ii) Eulerian, but not Hamiltonian

(iii) Hamiltonian, but not Eulerian (iv) neither Eulerian nor Hamiltonian.

2. Answer ***any three*** questions :

(a) Show that the graph contains no Euler circuit. 5

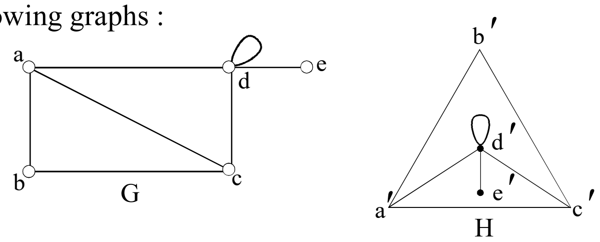


(b) Prove that the number of vertices of odd degree in a graph is an even number. 5

(c) Define a complete graph. Prove that a complete graph K_n with n vertices consists of ${}^{n}C_2$ number of edges. 1+4

(d) Let G be a simple bi-partite graph with e edges and n vertices. Prove that $e \le \frac{n^2}{4}$. 5

(e) Consider the following graphs :



Is G isomorphic to H? Justify. 5

3. Answer ***any four*** questions :

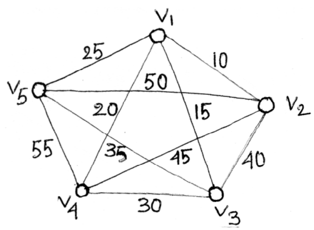
(a) (i) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

(ii) Prove that a graph containing a triangle cannot be bi-partite. 6+4

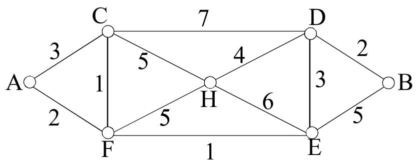
(b) (i) If a connected planar graph has n vertices and e edges, then prove that the number of regions in the graph is $e-n+2$.

(ii) Prove that Kuratowski's graph K_5 is non-planar. 6+4

(c) Define Hamiltonian cycle. For the following travelling salesman problem, find the shortest Hamiltonian cycle. 2+8

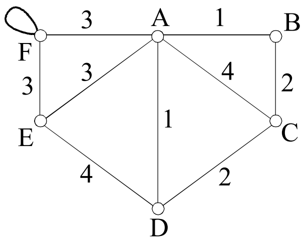


(d) Find the shortest distance between A and B using Dijkrasta's algorithm : 10

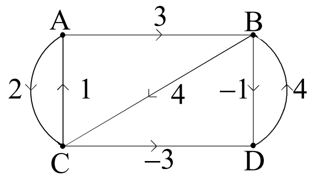


(e) (i) Prove that a graph G with n vertices is a tree iff G is connected and has $(n-1)$ edges.

(ii) Find a minimal spanning tree for the following graph : 5+5



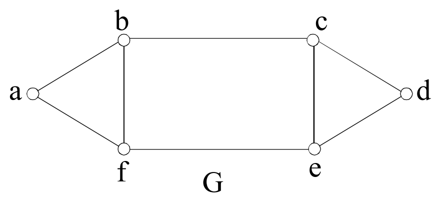
(f) Find the shortest distance matrix and the corresponding shortest path matrix for all the pairs of vertices in the following directed graph using Floyd–Warshall's algorithm. 10



(g) (i) Draw the graph of the following adjacency matrix :

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(ii) Determine the closure of the following graph G :



Conclude G is Hamiltonian or not. 5+5